The Quasi-Distributed Gap Technique for Planar Inductors-Design Guidelines

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Found in *IEEE Industry Applications Society Annual Meeting,* Oct. 1997, pp. 1147–1152.

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IEEE Industry Applications Society
Annual Meeting
New Orleans, Louisiana, October 5-8, 1997

The Quasi-Distributed Gap Technique for Planar Inductors: Design Guidelines

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Abstract -- The use of low-permeability magnetic material to form a uniformly distributed gap can facilitate the design of low-ac-resistance planar inductors. Alternatively, a quasi-distributed gap, which employs multiple small gaps to approximate a uniformly distributed gap, can effect the same result. Finite-element simulations are used to systematically analyze the design of quasi-distributed gap inductors. It is shown that a good approximation of a distributed gap is realized if the ratio of gap pitch to spacing between gap and conductor is less than four, or if the gap pitch is comparable to a skin depth or smaller. For a practical range of gap lengths, the gap length has little effect, but large gaps can reduce ac resistance. An analytic expression, which closely approximates ac resistance factor for a wide range of designs, is developed.

I. INTRODUCTION

Conductor losses in high-frequency magnetic components are strongly influenced by the magnetic field configuration and distribution in the winding area. For transformers, it is relatively straightforward to understand and to control the field and the resulting losses. However, in inductor designs, the field configuration is influenced by the geometry and position of the gap as well as the conductors. Avoiding excessive losses can be challenging.

Inductors in planar configurations are often desirable, either because of packaging constraints, or because of the fabrication technology. A planar configuration has the potential for particularly high ac conductor losses. This problem has been addressed extensively in the literature, e.g. [1, 2, 3, 4, 5, 6]. One of the most elegant solutions is to use a low-permeability material to effect a distributed gap across the top of the planar conductor [2], as shown in Fig. 1. Since high-performance, low-permeability power materials are often not readily available, the "quasi-distributed gap" [1] (shown in Fig. 2), which uses multiple small gaps to approximate a lower-permeability material, has been proposed as an alternative to the true distributed gap [6, 7, 4, 8, 9, 10].

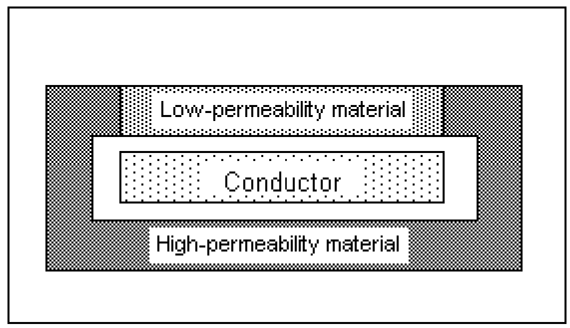

Figure 1. Distributed-gap inductor: use of a low-permeability material to achieve low ac resistance.

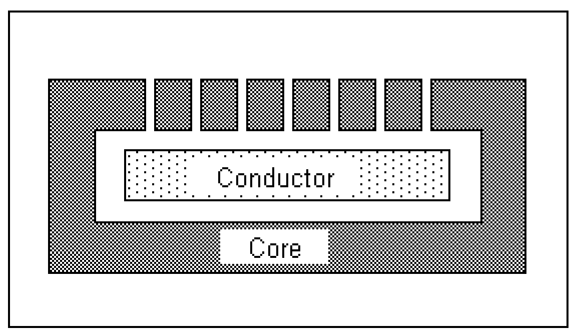

Figure 2. Quasi-distributed gap inductor using multiple small gaps to approximate a distributed gap

Although the principle of quasi-distributed gaps is well established, adequate design rules have not previously been developed. E.g., it is not immediately clear how many small gaps are necessary to approximate a distributed gap. For any given design, it is possible to use finite-element simulations to calculate losses, and use trial and error to find a design that works adequately. But this is not an efficient approach in practical design. It would be preferable to be able to calculate

[1] Denoted as a "discretely distributed gap" in [9]

or estimate the requirements for a quasi-distributed gap, to guide design without the need for repeated simulations.

An intuitive rule of thumb might be to space the conductor away from the gaps by a distance that is large compared to the gap length, to keep the fringing field from the gap away from the conductor. However, previous work has shown that this can result in losses nearly three times higher than losses with a true distributed gap [8]. Simulations of other designs show that losses can be within a few percent of the losses with a true distributed gap [6]. This intuitive rule of thumb is clearly inadequate. In this paper, a parametric study of losses is used to identify what the important relationships actually are. This results in a set of rules and analytic formulae that will allow designers to use a quasi-distributed gap to match the performance of a distributed gap.

A) Problem Definition and Simplification

We wish to reduce the problem to the minimum essential framework, to facilitate computations and conceptual understanding. To do this, we assume an infinitely wide quasi-distributed gap inductor. In this infinitely wide strip, with an infinite number of gaps, each gap is equivalent.[22] This means that we can simulate only a single gap, as shown in Fig. 3, with symmetry boundaries on the left and right.

We can describe this structure in terms of six geometric parameters as shown in Fig. 3: gap pitch p, gap length g, the space between the conductor and the gaps s, the thickness of the core t_m (assumed equal for top and bottom core sections), the thickness of the conductor t_{cu} and the spacing between the lower core and the conductor s_b. We normalize all these
dimensions in terms of skin depth in the conductor material,

$$\delta = \frac{1}{\sqrt{\pi \cdot f \cdot \mu_0 \cdot \sigma}}$$

, and assume the permeability of the magnetic material is infinite.

If the quasi-distributed gap approximates a distributed gap well, the choice of conductor thickness is already well understood: since current will mainly flow in the top skin depth, a thickness of one to two skin depths is sufficient to achieve near-minimum ac resistance. Thicker conductors can be used, but they will decrease only dc resistance. Thus, we perform most of our simulations with a conductor two skin depths thick. Since the field has decayed to near zero at the bottom of the conductor, the spacing to the back magnetic material, s_b, is not important. The thickness of the core material is also unimportant for the present purposes, as it is considered an ideal material. This leaves as parameters only g, s and p as shown in Fig. 3. By using a systematic approach to these variables, and by exploiting the symmetry of the problem to allow finite-element simulation of a section of length equal to only one gap pitch[33], it is possible to generate sufficient simulation data to understand the problem thoroughly.

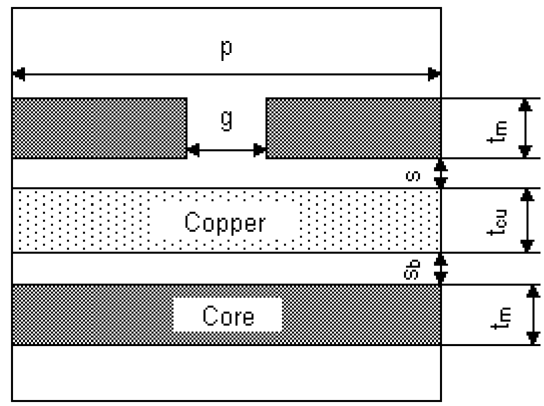

Figure 3. A section of a quasi-distributed gap used for simulation. Dimensions are normalized to one skin depth in the conductor.

II. SIMULATION RESULTS AND ANALYSIS

A) Simulation Results

The results of simulations are plotted Fig. 4, which shows the ac resistance factor

$$F_r = R_{ac} / R_{dc}$$

for various values of gap pitch p and spacing between gap and conductor s. To explain the results qualitatively, we first consider the variations in the pitch. As the gap pitch gets larger, F_r increases significantly. This is due to the tendency for current to crowd near the gaps, as shown in Fig. 5. As the gap pitch gets smaller, the region of current crowding becomes a larger fraction of the overall width. When adjacent regions overlap (p equal to one to two skin depths), the current distribution becomes approximately uniform, as shown in Fig. 6. The ac resistance factor approaches a minimum, almost equal to the ac resistance factor with a distributed gap.

[2]2 An infinitely wide strip like this is an ill-posed problem, in that the inductance per unit width becomes undefined, but this is not important to the results here.

[3]3 One could use the lateral symmetry of the structure in Fig. 3 to further shorten simulation time, but this is not convenient with the software tool we used.

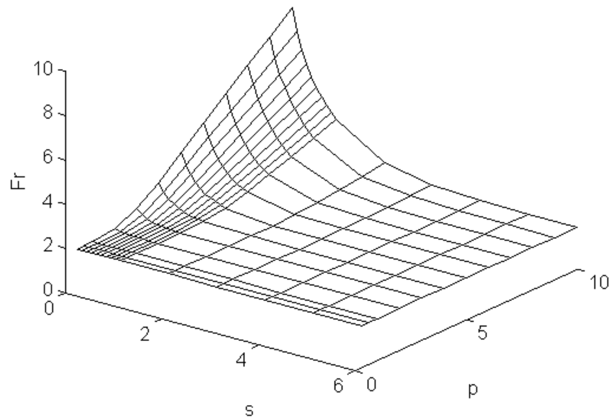

Figure 4: Simulated ac resistance factor, F_r as a function of gap pitch *p*, and the conductor-gap spacing *s*, both normalized to skin depth. The gap length was 0.1 (one tenth of a skin depth) for these simulations; however, the results apply for any small gap.

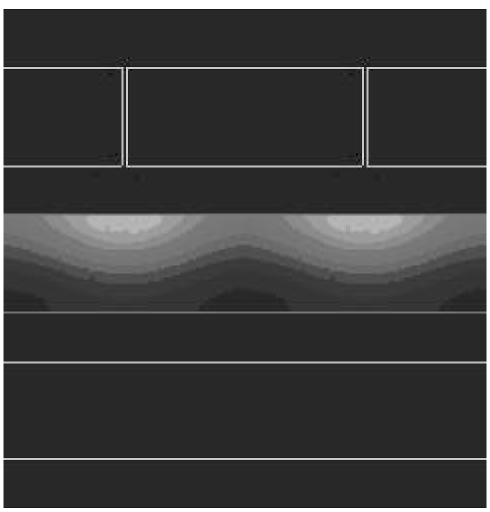

Figure 5: Current distribution in the cross-section (p = 5, s = 1, g = 0.1).

Spacing the gaps away from the conductor can also be beneficial for decreased ac losses. In this sense, intuition regarding gap fringing fields is correct. The distance required for any given ac resistance is affected by gap pitch, *p*, as can be seen in Fig. 4. One may think of this as the myopia of the eddy current losses. If the gaps are far enough away, they "blur out" and "look" like a single distributed gap.

In the above discussions, the gap length *g* is fixed. In order to illustrate the effect of gap length on ac losses, two typical values of gap pitch and spacing from the region shown in Fig. 4 are selected. The first one represents the case where original value of F_r is relatively low, while the second one represents the case

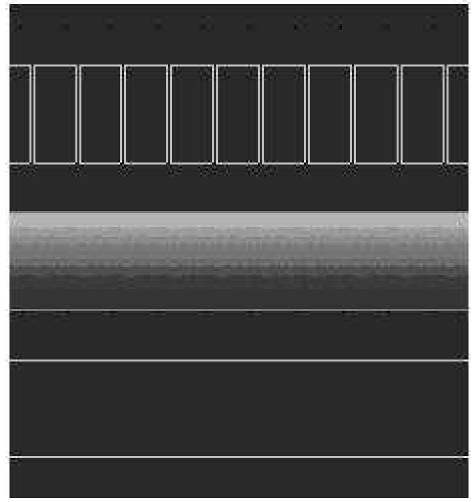

Figure 6: Current distribution in the cross-section (p = 1, s = 1, g = 0.1).

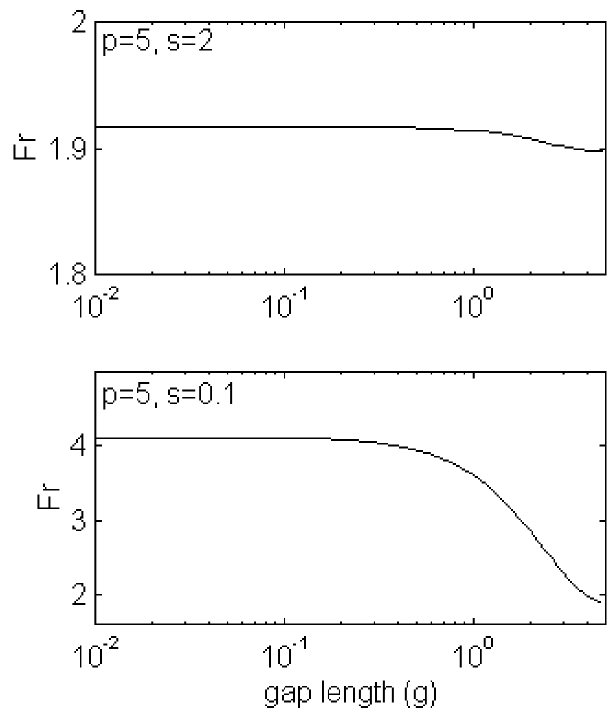

Figure 7. Ac resistance factor, F_r as a function of gap length, *g*, normalized to one skin depth, for two geometric configurations defined by the normalized values of *p* and *s* indicated (see Fig. 3).

of a higher original F_r value. The simulation results for these two cases with a range of gap lengths are shown in Fig. 7.

To explain the results shown in Fig. 7, we return to the idea of current crowding into the region near the gaps. As the gap gets bigger than one skin depth, the width of the gap becomes the dominant factor determining the width of the region of current crowding. Thus a wider gap can spread the current out further and decrease losses. This tendency can be seen by comparing Fig. 8 to Fig. 5.

Although wide gaps can reduce ac losses, this is not useful in most practical situations. The inductance requirement will typically constrain the total gap length, *g(*, to be a fixed, small fraction of the conductor width *w*. Suppose *n* gaps are distributed within *w*. In this case, *g* = *g(/n* and *p* = *w/n*. To get ac losses near those of a true distributed-gap inductor, one might think that *n* should be small in order to make the gap length larger. However, if *n* is reduced, the gap pitch will also become larger. As shown in Fig. 4, making the gap pitch larger will increase F_r dramatically.

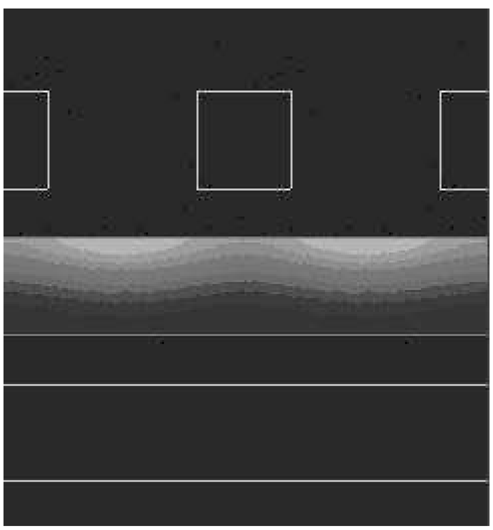

Figure 8 Current distribution along the cross-section (*p* = 5, *s* = 1, two gaps, *g* = 3).

For the example shown in Fig. 7(a), changing the gap length can not improve F_r significantly. In the example shown in Fig. 7 (b), in order to significantly reduce losses and achieve F_r of about 2.5, the gap length has to be about 50% of the pitch length. This results in an effective permeability of the quasi-distributed gap of about 2, too low for most inductor designs. In practice, gap length *g* is typically constrained to be small enough that it has little effect on the ac resistance.

Fig. 7 also shows that gap length has almost no effect on ac resistance in the region of small gaps. E.g., in Fig. 7 (b), a factor of 30 change in *g* (from 0.01(to 0.3(, where *(* is the skin depth in the conductor) results in only about a 1.5% reduction in F_r. Note that this includes a range of ratios of spacing to gap length from *s/g = 10* to *s/g = 0.33*, confirming that a rule-of-thumb based on this ratio is not useful.

Most of our simulations use a small gap, g = (/ 10, and the results are applicable to any design with a gap that is small compared to a skin depth, although very large gaps can reduce losses somewhat relative to the results we report below. With small gaps, and fixed conductor thickness, F_r may be described as a function of just two variables, the spacing from the gap to the conductor *s* and the gap pitch *p*. It is the size of dimensions relative to skin depth that really matters; thus, the data in Fig. 4, where the dimensions are normalized to one skin depth, can be used to determine the ac resistance for any design. (They directly apply only if the thickness of the conductor is equal to two skin depths. Scaling for other thickness of conductor will be discussed later.)

Both in Fig. 4 and Fig. 7, the minimum ac resistance factor is about $F_r = 1.9$. A one-dimensional solution of Maxwell's equations yields the following expression for ac resistance factor with a uniformly distributed gap [11], [12]:

$$F_r = \frac{2 \cdot t_{cu} \cdot \sinh(\frac{2 \cdot t_{cu}}{\delta}) + \sin(\frac{2 \cdot t_{cu}}{\delta})}{\delta \cdot \cosh(\frac{2 \cdot t_{cu}}{\delta}) - \cos(\frac{2 \cdot t_{cu}}{\delta})} \quad (1),$$

where t_{cu} is the thickness of the conductor. For $t_{cu} = 2 \cdot \delta$, $F_r = 1.8978$, very close to the minimum ac resistance factor of the simulation results.

B) Analytical Approximation

In order to facilitate design without the need to use tables or plots of data, an empirical expression to describe F_r as a function of conductor-gap spacing *s* and gap pitch *p* is in need. Using a numerical least-square fit, we developed the following expression to approximate $F_r(s,p)$:

$$F_r(s,p) = \frac{-k}{(b^{-n} + p^{-n})^{1/n}} + k \cdot p + 1.9 \quad , \quad (2)$$

where $n = 5.4$, (3)

$$k = \frac{0.95}{0.95 + 1.4 \cdot s} , \quad (4)$$

$$b = 3.33 \cdot s + 2.14 . \quad (5)$$

The F_r values computed by the above expressions are compared to simulation results in Fig. 9. It can be seen that this expression approximates the simulation results very well, with relative error less than 4.5%, and absolute error in F_r less than 0.08. This expression also remains accurate for configurations with large *s* and *p*, as shown in Fig. 10.

For large s, (3) and (4) become:

$$k \cong \frac{0.68}{s} \quad (6)$$

$$b \cong 3.33 \cdot s \quad (7).$$

We can rewrite the expression (1) as:

$$Fr = \frac{-0.68}{\left(3.33+\left(\frac{p}{s}\right)^{-n}\right)^{1/n}} + 0.68\cdot\left(\frac{p}{s}\right)+1.9 \quad (8).$$

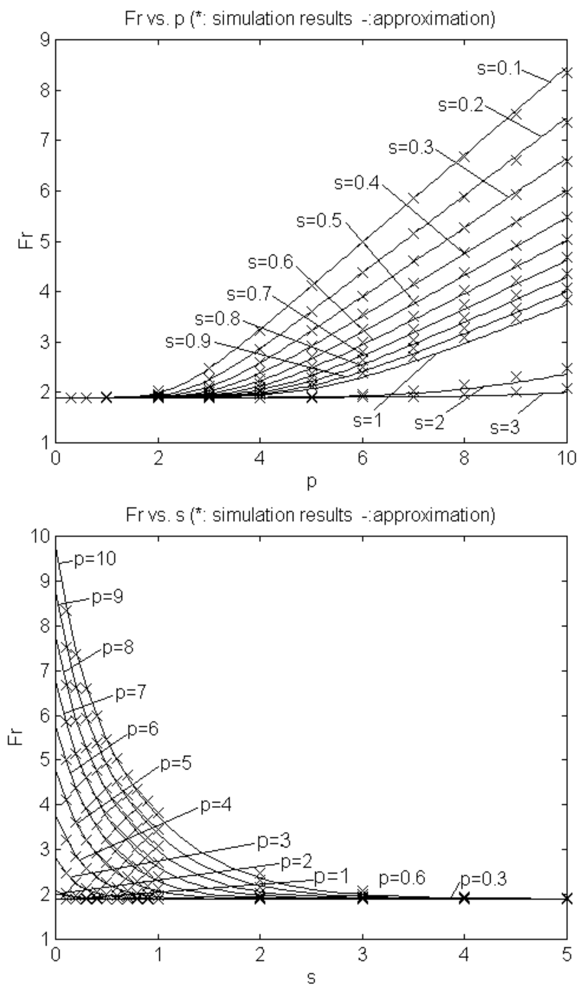

Figure 9: Comparison between values of F_r computed by using the approximation expression (2) and values obtained from finite-element simulations.

If the ratio of $\frac{p}{s}$ is much greater than 3.33, the equation will enter a roughly linear region and it can be simplified as $Fr = 0.68\cdot\frac{p}{s} - 0.36$. In contrast, If the ratio of $\frac{p}{s}$ is much smaller than 3.33, the equation will enter a constant region with *Fr* = 1.9. These two trends are shown in Fig. 10. In a practical design, if the space is not critical, we can thus make the spacing s relatively larger, like about one fourth of a gap pitch, to lower the ac power losses. From Fig. 10, we also can conclude that the expressions (1), (2), (3), (4) is still a quite accurate model of *Fr* even for large conductor-gap spacing.

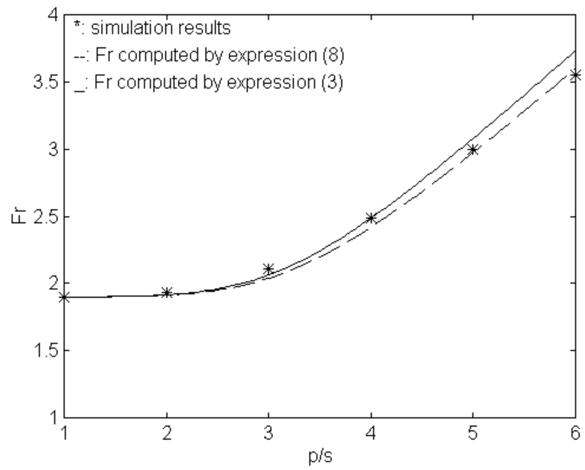

Figure 10: F_r as a function of the ratio *p/s* with *s* = 20. The results are approximately the same for any large value of *s*.

C) Full Device vs. Periodic Segments

To check the assumption that a complete device with multiple gaps could be modeled by a single segment of a periodic structure, we simulated two-dimensional cross sections of complete devices, as shown in Fig. 11. Some simulations used a symmetry boundary on one side to represent a device in which the current flows in a planar loop, returning in a adjacent repetition of the structure in Fig.11. In all these simulations, the ac resistance factor was within 2% of the value obtained from the simulation of a single segment. We conclude that in typical practical designs, the losses are accurately modeled by a single segment.

D) Finite Permeability of Core Material

Throughout our simulations we assumed that the permeability of the core material is almost infinite, which implies that the reluctance of the core is zero and flux is only determined by the reluctance of gap. In a real design, if we consider the effect of the finite reluctance of the core, the fraction of the MMF drop across the core will be higher, while the MMF drop across the gaps will be lower. The MMF drop across the core is similar to the MMF drop in a true distributed gap, and thus, the ac resistance is reduced. In order to illustrate this quantitatively, simulations using finite-permeability core material are performed. For a configuration of p = 5 and s = 1,

$$Fr|_{\mu'=\infty} \cong 2.13,$$

$$Fr|_{\mu'=1000} \cong 1.85,$$

where μ' is the relative permeability of core material. These results confirm the previous discussion. Note that the ac resistance factor with $\mu' = 1000$ is less than 1.9, the minimum with a distributed gap as shown in Fig. 1. This is because with low permeability material underneath the conductor, we start to use the bottom surface as well as the top surface to conduct ac current.

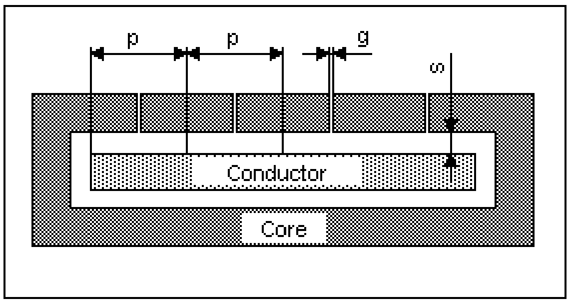

Figure 11: A full device with four equally distributed gaps.

E) Thickness of the Conductor

In the discussion above the thickness of the conductor is fixed to be two times the skin depth. Although increasing the thickness of the conductor will lower the dc resistance, it will not improve the ac resistance significantly. Given a thickness larger than two skin depths, we can use (9) to estimate F_r.

$$F_r(t = x) \cong F_r(t = 2) \cdot \frac{x}{2} \qquad (9),$$

where the thickness of the conductor x is greater than two skin depths.

III. DESIGN RULES

Based on the above analyses, we can glean the following design rules. Low ac losses (ac resistance factor less than 2.5(approaching that of a distributed-gap inductor), can be obtained if either of the following conditions are met:

The ratio of gap pitch to conductor-gap spacing (*p/s*) is less than four, or

The gap pitch *p* is less than 2.5 times skin depth.

If the gap length *g* is sufficiently large, it can also reduce the ac resistance. However *g* must be a substantial fraction of the gap pitch *p* to have tangible effect on ac resistance. This is unlikely to be practical in most designs.

These results can be applied to designs using a wide range of fabrication technologies and frequencies. For example, the results could apply to ferrite-core inductors with mm-scale dimensions used at high frequencies, or inductors fabricated with thin-film deposition and photolithography, with (m-scale dimensions used at MHz frequencies. By using (2), the approximate F_r value can be found to estimate the configuration of any design.

IV. CONCLUSION

Simulation of a single segment of a periodic quasi-distributed gap inductor is adequate to predict the ac resistance. A set of such simulations produces data that can be used for a wide range of designs. Approximate analytic formulae that describe these data accurately have been developed. This has led to a simple set of design rules that can be used to ensure a design which will have low ac resistance.

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