General Comparison of Power Loss in Single-Layer and Multi-Layer Windings

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Abstract **This paper systematically and generally addresses the question of when a single-layer winding can have lower loss than a multi-layer winding. Two cases for single-layer windings are investigated. First, when very high frequencies require thinner conductor layers or thinner wire than is practical. Second, when the current waveform contains substantial harmonics. We conclude that multi-layer windings typically provide lower loss except when layer thickness constraints prevent the use of a thickness-to-skin-depth ratio less than 1.45. However, with substantial harmonic content, the advantage of multi-layer windings is typically small and is not always worthwhile in practice.**

I. Introduction

A well-established approach to reducing eddy-current losses in windings is to configure conductors in many layers that are each small compared to a skin depth. This can be done with thin foil, thin printed circuit windings, fine wire, or fine-stranded litz wire (in litz wire, the strands are not neatly organized into layers, but the effect on loss is similar). Although having layers thin compared to a skin depth is beneficial, having many layers can increase loss, because the field in a given layer is the sum of the fields contributed by the many other layers. Thus, this approach can actually lead to increased losses if the winding is poorly designed. Although the eddy-current losses can be small, they increase in proportion to frequency squared. This effect is well understood and addressed in the literature, and methods for optimizing multi-layer windings have been developed to avoid incurring excessive loss. For example, optimization of foil windings is addressed in [1], optimization of solid-wire winding in [2], and optimization of litz-wire windings in [3] [5].

In some cases, however, a single-layer winding may be better than any multi-layer winding. In a single-layer winding with high-frequency current, the current flows on the surface in a layer one skin depth deep. Because there are no other layers inducing a field, the loss depends only on the resistance of the layer where current flows, and resistance is inversely proportional to skin depth, and thus is proportional to the square root of frequency.

Given the f^2 dependence of loss in a multi-layer winding (which holds when layers are small compared to skin depth) and the $\sqrt{f}$ behavior of loss in a single-layer winding, one can conclude that, for a given pair of windings, it is likely that above some frequency, the single-layer winding is superior.[1] However, one cannot conclude that therefore a single-layer winding is in general superior for high frequencies, because, for any given high frequency, making the layers thinner in the multi-layer winding can reduce high-frequency losses arbitrarily.

But there are two arguments in favor of single-layer windings that cannot be dismissed so readily. Firstly, very high frequencies may require thinner conductor layers or thinner wire than is practical. Secondly, the current waveform may not be a single frequency: it may contain harmonics and/or a dc component. Even if the design is optimized for low fundamental-frequency resistance, the harmonics may incur substantial loss if the loss increases in proportion to frequency squared. Of course, the design should be optimized for minimum total loss, considering the actual waveform(s), as discussed in [1], [5], but this still entails a compromise and it is plausible that in some cases with high harmonic content a single-layer winding is preferred. For waveforms that include substantial dc or low-frequency content, multi-layer windings have two additional disadvantages. One is that as the number of layers is increased, the packing factor suffers. Litz wire often has particularly poor packing factor, although rectangular compacted litz wire [6] can mitigate this. Another is that increasing the thickness of layers to decrease dc resistance can increase ac loss, whereas in single-layer designs, there is no high-frequency penalty for increasing the layer thickness.

The purpose of this paper is to systematically and generally address the question of when a multi-layer winding can have lower loss than a single-layer winding. In Section II the problem to be addressed will be more precisely defined, and the loss models used will be introduced. In Section III, the case of practical limits on thickness will be addressed. In Section IV, the effects of non-sinusoidal waveforms will be addressed.

II. Problem Definition and Loss Models

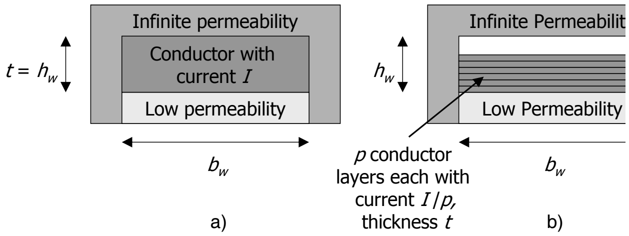
A. Parameter Definitions

We define ac resistance in terms of the loss in a winding P_w as

$$R_{ac} = P_w / I_{ac,rms}^2 \quad (1)$$

where $I_{ac,rms}$ is the rms current in the winding. R_{ac} therefore depends on the current waveform. Considering sinusoidal waveforms allows R_{ac} to be defined as a function of frequency. In a transformer, R_{ac}, defined as in (1), would depend on current waveforms in all windings in the transformer, because of their effects on fields and therefore losses in all windings. Such mutual resistance effects indicate a resistance matrix with non-zero cross terms [7]. Because we use (1), the analysis in this paper is limited to inductors and two-winding transformers with the same current

[1] See the Appendix for a detailed explanation.



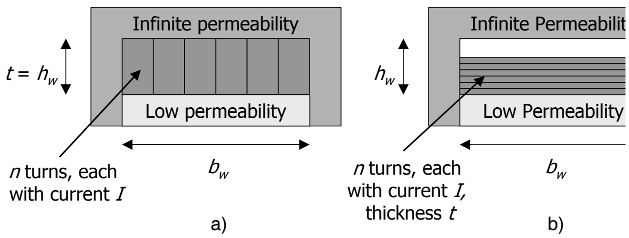


Fig. 2. A comparison of practical single-layer and multi-layer windings (a) Single-layer ($p = 1$) edge-wound foil with six turns ($N = 6$). (b) Barrel-wound foil with the same number of turns ($N = 6$), but with the number of layers equal to the number of turns ($p = N$). All turns are in series in both cases.

waveform in both windings (scaled by the turns ratio). Despite this limitation, we expect this paper to provide insight that is valuable for other cases, and to lay the groundwork for analysis including mutual resistance effects.

B. Problem De nition

For the purpose of this analysis, the multi-layer and single-layer cases will be abstracted to be those illustrated in Fig. 1: a two-dimensional cross section of a single-layer distributed-gap inductor with a single thick turn of conductor, carrying current I, and the same core with a single-turn conductor divided into p layers of conductor each carrying current I/p. This simple con guration does not directly correspond to any practical situation, but it can represent many different practical situations, with varying degrees of approximation, as discussed below.

C. Application to Practical Winding Types

The con guration in Fig. 1 is not practical because, rstly, a single-turn winding is rarely practical, and, secondly, because if a single-turn winding were to be divided into parallel layers, the current would not split equally between the layers. However, the con guration in Fig. 1 can be related to particular design situations.

One particular practical situation that is very similar to Fig. 1 is a comparison of a multi-turn barrel-wound foil winding and an edge-wound winding [8], as shown in Fig. 2. In this case, the number of layers in the barrel-wound foil (Fig. 2b), p, is equal to the number of turns, N. Both windings have the same ac resistance factor, F_r, as the corresponding winding in Fig. 1, and their ac resistances are the ac resistances of the corresponding windings in Fig. 1 scaled up by a factor of N^2.

By breaking each layer in Fig. 2 into multiple turns, it is possible to decouple the number of turns from the number of layers, as shown in Fig. 3. Windings similar to this may be wound with rectangular magnet wire or constructed with printed circuit board (PCB) technology, including planar windings using conventional substrates or windings using folded or wound e xible substrates. In order to ensure equal current in each conductor segment, this strategy can only be used to obtain a number of layers smaller than the number of turns.

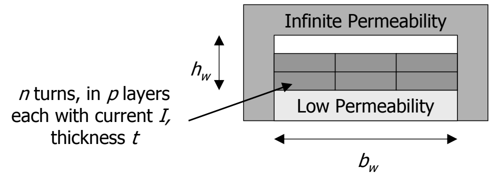


Fig. 3. Modi ed version of Fig. 2b with different numbers of turns and layers: six turns ($N = 6$) and two layers ($p = 2$) All turns are in series.

It is common to analyze a winding wound with round wire by approximating it with layers of foil, an approach usually attributed to Dowell [9]. Thus, it is straightforward to consider wire windings in the same framework. Recent work has shown that the accuracy of the standard approximation is poor in some cases [10], but improved models [11] show the same general trends as the simple Dowell analysis. Thus, general conclusions about when multi-layer or single-layer windings are preferred from the analysis of Fig. 1 will hold for wire windings as well.

In wire windings, litz wire can be used to overcome the dif culties with paralleling conductors, if the strands of ne insulated wire that constitute the litz conductor are twisted together in a true litz pattern such that each strand moves between positions in the bundle. In this case, there is no distinction between strands that would lead to more current in one or less in another, and thus the current will be shared equally between strands. Litz wire windings do not have neatly structured layers at the strand level, but in most situations this does not signi cantly affect the loss [4].

D. Application to Practical Inductors and Transformers

Figs. 1 3 show an inductor with a distributed gap. Although a distributed gap can be a practical way to achieve a one-dimensional eld, it is not very widely used in practice, and the practical cases of most interest are transformer windings and inductors with conventional lumped (discrete) air gaps. In a standard two-winding transformer with layered construction and an ungapped high-permeability core, the MMF produced by one winding is dropped by another winding, just as the MMF produced by the winding in Figs. 1 3 is dropped across the distributed gap. Thus, the loss analysis is the same for a simple transformer and the results of analyzing the con guration in Fig. 1 apply directly.

E. Models

For the situations shown in Fig. 1, the eld is one-dimensional, and the Dowell model is an exact solution of Maxwell's equations. The ac resistance factor can be expressed as [12]

$$\frac{R_{ac}}{R_{dc}} = \Delta\Big[\frac{\sinh 2\Delta + \sin 2\Delta}{\cosh 2\Delta - \cos 2\Delta} + \frac{2(p^2-1)}{3}\frac{\sinh \Delta - \sin \Delta}{\cosh \Delta + \cos \Delta}\Big] \quad (2)$$

where Δ is the ratio of layer thickness to skin depth $\delta = \sqrt{\frac{\rho}{\pi\mu f}}$ where ρ is the conductor resistivity, μ is the conductor permeability and f is the frequency of a sinusoidal current. For small values of Δ, as are typically advantageous in a multi-layer winding [1], this can be approximated as [13]

$$F_r = \Big[1 + \Big(\frac{5p^2-1}{45}\Big)\Delta_{ml}^4\Big] \quad (3)$$

where Δ_{ml} is the value of Δ for a multi-layer foil winding.

For large values of Δ, it is possible to calculate the loss based on the assumption that the current flows in a layer one skin-depth thick. In a single-layer winding, the ac resistance factor is then

$$F_r = \Delta_{sl} \quad (4)$$

where Δ_{sl} is the value of Δ for a single-layer foil winding. Losses with non-sinusoidal current waveforms can be analyzed using Fourier analysis. A more detailed description of how to use Fourier analysis for non-sinusoidal waveforms is provided in Section IV.

III. Case 1: Minimum Thickness Constraint

In the case of a sinusoidal winding current, a multi-layer winding may, in theory, be made to have arbitrarily low eddy-current losses through the use of a large number of thin layers. Thus, a multi-layer winding is the clear winner in this situation, but this configuration may not be feasible in practice. One constraint is the practicality of very thin layers. For example, litz wire is readily available commercially with strands as fine as 48 AWG (32 μm), but finer strands are difficult to manufacture or to obtain commericially. Thus, an important case to analyze is with a fixed minimum layer thickness.

With a fixed minimum layer thickness, the lowest loss is, in some cases, obtained without the bobbin completely filled, so to find the lowest loss case we must find the optimum number of layers p. The dc resistance is inversely proportional to the number of layers, so the loss is proportional to

$$\frac{F_r}{p} = \Big[1 + \Big(\frac{5p^2-1}{45}\Big)\Delta_{ml}^4\Big]\Big(\frac{1}{p}\Big) \quad (5)$$

Setting the derivative of (5) with respect to p equal to zero results in the following optimum value of p

$$p_{opt} = \sqrt{\left(\frac{9}{\Delta_{ml}^4} - \frac{1}{5}\right)} \quad (6)$$

For large values of p (small values of Δ_{ml}), (6) becomes

$$p_{opt} = \frac{3}{\Delta_{ml}^2} \quad (7)$$

Now the loss of a single-layer winding can be compared to that of a multi-layer winding by relating R_{dc} for a single-layer to R_{dc} for a multi-layer design and then multiplying by the corresponding F_r. R_{dc} for a multi-layer winding is

$$R_{dc} = \frac{k}{pt} \quad (8)$$

where t is the layer thickness and $k = \frac{N\ell_t\rho}{b_w}$. In this equation, N is the number of turns, ℓ_t is the mean length turn of the winding, ρ is the resistivity of the conductor, and b_w is the breadth of the winding window. R_{dc} for a single-layer winding is

$$R_{dc} = \frac{k}{h_w} \quad (9)$$

where h_w is the height of the full winding window. Substituting (7) into (3) results in an ac resistance factor of $F_r = 2$ for the optimized multi-layer design.

F_r for a single-layer design is given by (4). The product of F_r and R_{dc} is proportional to winding loss and results in

$$P_{sl} \propto \frac{k}{\delta} \quad (10)$$

for a single-layer design. For a multi-layer design (7) is substituted into (8) and then multiplied by F_r resulting in

$$P_{ml} \propto \frac{k}{\delta}\frac{2}{3}\Delta_{ml} \quad (11)$$

Finally, the loss ratio between a multi-layer and single-layer design is then given by

$$\frac{P_{ml}}{P_{sl}} = \frac{2}{3}\Delta_{ml} \quad (12)$$

This equation can then be used to easily determine if a multi-layer or single-layer foil winding design should be used, based on the thickness of foil available for the given design.

To express (12) for round wire we need to find the correct layer thickness to use in the approximation of the Dowell model (2), given a wire diameter d. The correct thickness is given in the appendix of [4] as

$$t = \left(\frac{3\pi}{16}\right)^{\frac{1}{4}} d_c \quad (13)$$

Substituting (13) into (12) we find that the loss ratio for round wire is

$$\frac{P_{ml}}{P_{sl}} = \frac{2}{3}\left(\frac{3\pi}{16}\right)^{\frac{1}{4}}\frac{d_c}{\delta} = 0.584\left(\frac{d_c}{\delta}\right) \quad (14)$$

The ratios in (12) and (14) verify that the multi-layer winding design has lower loss than a single-layer winding design for small values of Δ_{ml} and $\frac{d_c}{\delta}$. It also provides a quick and easy way to evaluate the improvement possible using a given technology that has a set layer thickness. For example, 32 μm (AWG 48) litz wire can provide a significant (factor-of-two) improvement over single-layer designs for up to 4 MHz, based on (12) and the analysis of litz wire in [4].

The above analysis was based on a simplified model. The optimal number of layers (p_{opt}) can also be found using the full

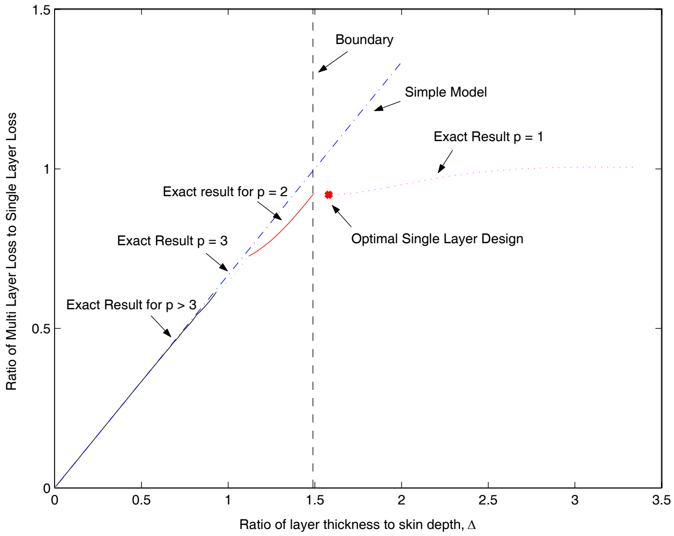


Fig. 4. Comparison of loss with a thick single-layer design to loss with an optimized multi-layer design, with the layer thickness constrained to the value on the horizontal axis. To the left of the vertical dotted line, multiple layers are used; to the right only a single layer. The large black dot is loss of a single layer design using the optimum thickness. The dash-dot line shows the simple model of (12), which works best for small Δ but works well for the full range where multi-layer designs are useful, up to $\Delta = 1.49$.

Dowell expression (2) to more accurately compare single-layer and multi-layer designs. Fig. 4 is a comparison of (12) to designs that are numerically optimized using Dowell's analysis.

Fig. 4 compares the loss of a thick single-layer design to that of a multi-layer lowest loss design using the optimum number of layers. Fig. 4 con rms that (12) is accurate for $\Delta_{ml} \leq 1.49$. The model error for $p \geq 5$ is less than 1%. The error is less than 1.8% for $p > 4$, less than 3.4% for $p > 2$, but increases to 8.6% over the range of designs for which the optimal value of p is 2.

IV. Case 2: Effect of Harmonics and DC Components

Non-sinusoidal winding currents, as are common in power electronics, have substantial harmonic components. In a multi-layer design, ac resistance can be proportional to the square of frequency (3), whereas it is only proportional to the square root of frequency in a single-layer design (4). Thus, we might expect single-layer designs to be advantageous for waveforms with large harmonic content. This is often considered to be a disadvantage of litz wire, and some designers avoid litz wire for designs with large harmonic content.

To evaluate this hypothesis, we wish to compare single-layer and multi-layer designs, using the optimum layer thickness for any given number of layers. In [1], a solution for the optimum thickness with a non-sinusoidal waveform is found based on (3) with the Fourier series expansion of the current waveform, and this solution is conveniently expressed in terms of rms values of the waveform and its derivative. It is shown that (3) is an adequate approximation for (2) in most waveforms that have typical amounts of harmonic content.

As shown below, the only cases in which single layer windings might be superior to multi-layer windings are extreme cases with

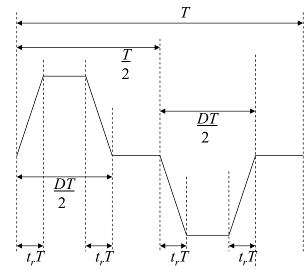


Fig. 5. Bipolar PWM waveform with a period T, rise time fraction t_r (and thus rise time t_rT), and duty cycle D.

high harmonic content. Although the analysis in [1] based on (3) is adequate for most practical design work, we wished to undertake a more careful examination of these cases in which (3) may not be accurate. Thus, we calculated loss by combining Fourier analysis with the Dowell model (2), and found the optimum thickness for minimum loss at a given number of layers through an automatic search process implemented in software. The calculated loss value is then normalized to the minimum loss with one layer.

A. Bipolar PWM waveform

A bipolar PWM waveform with a pulse of width $\frac{DT}{2}$ and a rise time fraction of t_r as shown in Fig. 5 was investigated rst. The curves in Fig. 6 show the minimum loss for any given number of layers, normalized to the minimum single-layer loss. The solid-line curve in Fig. 6 is for the bipolar PWM waveform (Fig. 5) with $D = 50\%$ and $t_r = 1\%$. It shows that winding losses are lower using multi-layer designs. Numerical experiments showed that this is true for most practical parameter values for this waveform (Fig. 5). To determine whether this is always true, we searched for a counterexample. The dashed line curve in Fig. 6 is for a waveform with $D = 26\%$ and $t_r = 0.01\%$. This waveform has increasing loss for numbers of layers greater than one. Although this increase occurs only up to 1.4 layers, not until 3.5 layers does the loss for a multi-layer design become better than that of a single-layer design. However, the best multi-layer design for a given number of layers is never much worse than a single-layer design. At the point where a single layer design has the biggest advantage (1.4 layers), it is only 2% better than a multi-layer design.

Fig. 7 describes combinations of parameters t_r and D for which a single-layer design may be better than a multi-layer design. The number of layers needed for a multi-layer design to achieve lower loss than a single-layer design is listed on the contours of the plot. For the range shown on the plot (rise time fractions down to 0.01%), designs with three or more layers can always outperform single-layer designs, and for most cases even fewer layers are required. Thus, we can tentatively conclude that multi-layer designs are in fact generally superior to single-layer designs.

However, the improvement provided by a multi-layer design may not always be signi cant. The dashed-line curve in Fig. 6 shows only a slight improvement, even at 10 layers. The cost

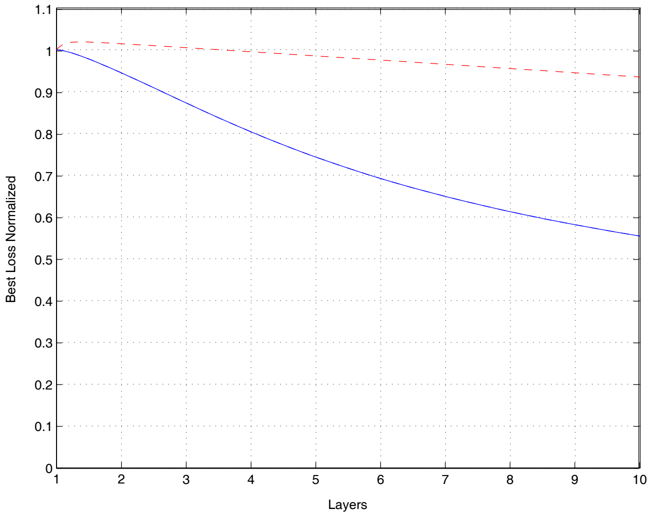


Fig. 6. Minimum loss for any given number of layers, normalized to the minimum single-layer loss, for two PWM waveforms. The solid line is for a bipolar PWM waveform with a duty cycle of 50%, a rise time fraction of 1% (thus having typical harmonic content). The dashed line is for the waveform with a duty cycle of 26%, a rise time fraction of .01%, (thus having high harmonic content).

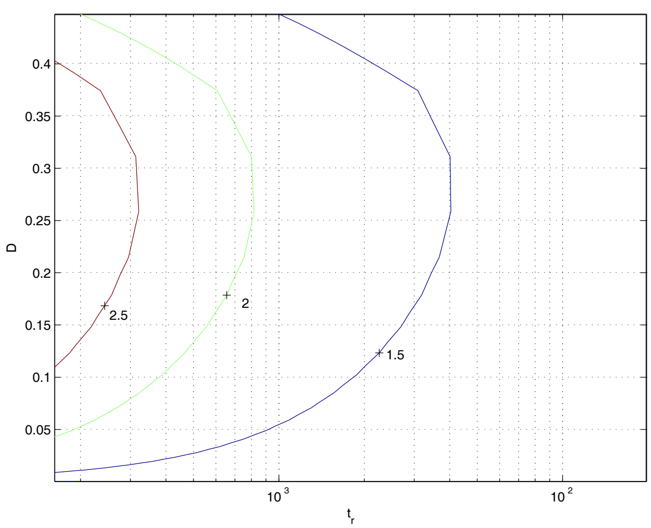


Fig. 7. Contour plot of number of layers necessary for a multi-layer design to have lower loss than a single-layer design for a bipolar PWM waveform with the indicated rise time fraction (x-axis) and duty cycle (y-axis).

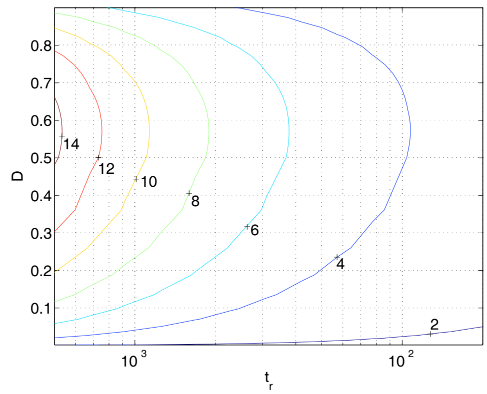


Fig. 8. Number of layers needed to achieve 20% reduced loss versus rise time fraction (x-axis) and duty cycle (y-axis) for a bipolar PWM waveform.

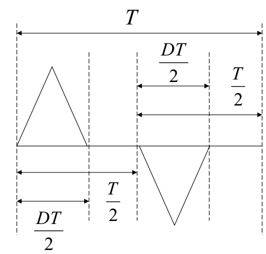


Fig. 9. Bipolar triangle waveform with a period T and duty cycle D.

of a large number of layers may be signi cant, and a signi cant improvement in loss would be needed to justify it. To investigate this issue, we arbitrarily chose 20% improvement as the threshold of signi cant loss improvement. Fig. 8 shows contour lines on similar t_r and D axes as Fig. 7, but with the contour lines showing the number of layers required to effect a 20% loss reduction. The number of layers required is highest for moderate duty cycles and small rise times; about four layers for 1% rise time and about ten layers for 0.1% rise time. This analysis makes it clear that achieving substantial loss reduction for waveforms with strong harmonic content requires many layers. While multiple layers may still be the best choice for very low loss, the simplicity and low cost of single-layer designs makes them attractive, given that their loss may be only slightly higher.

B. Other waveforms

The analysis of number of layers required to produce a 20% reduction in loss was repeated for all the common power electronics waveforms tabulated in [1]. Similar results were observed for groups of similar waveforms all the square waveforms produced results similar to those in Fig. 7. A triangular waveform (such as the ac current in the lter inductor of a PWM dc-dc converter) typically requires two to three layers for a 20% reduction in loss, but can require more layers for extreme duty cycles: four for 12% duty cycle, 10 for 5% duty cycle, or even more at smaller duty cycles, as shown in Fig. 10. Results for the bipolar triangular pulse waveform shown in Fig. 9, along with results for similar unipolar waveforms and waveforms with half-cycle sinusoidal pulses, are shown in Fig. 11. Of these, the Fig. 9 waveform requires the largest number of layers: a remarkably constant 4.5 layers for a wide range of duty cycles.

We can summarize the results of examining different waveforms as follows: Obtaining a 20% reduction in loss relative to a single-layer winding requires four or more layers for:

- The bipolar triangular pulse waveform shown in Fig. 9 for most duty cycles.
- Triangle waveforms with duty cycles of 12% or less.
- Square waveforms with rise times of about 1% or less.

Obtaining a 20% reduction in loss relative to a single-layer winding requires ten or more layers for:

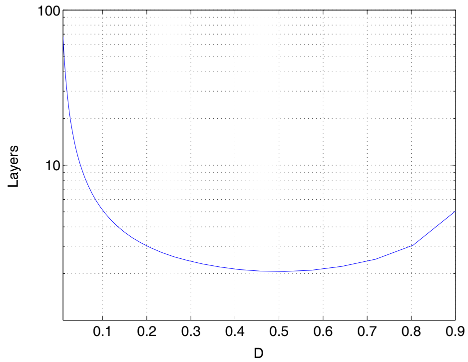


Fig. 10. Number of layers needed to achieve 20% reduced loss versus duty cycle for a triangular waveform.

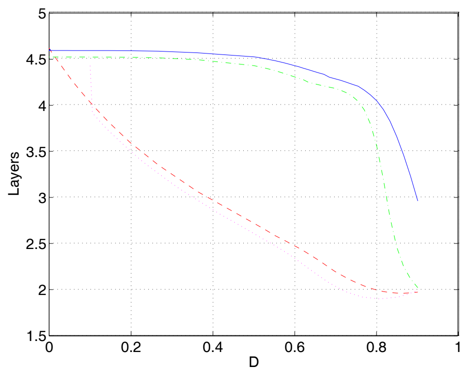


Fig. 11. Number of layers needed to achieve 20% reduced loss versus duty cycle for triangular and half-cycle sinusoidal pulse waveforms. The unipolar representations of these waveforms are the dashed and dotted curves. The bipolar representations of these waveforms are the solid and dot-dash curves.

- Triangle waveforms with duty cycles of 5% or less.
- Square waveforms with rise times of about 0.1% or less.

C. Waveforms with two frequency components

To better understand the impact of different harmonic frequencies and amplitudes, we considered a waveform that is the sum of two sine waves. Again, we considered designs with the layer thickness chosen for minimum loss for a given number of layers. Normalized minimum loss was plotted as a function of number of layers for a wide range of ratios between the amplitudes and frequencies of the two sinusoids in the waveform. In general three types of curves were observed. Fig. 12 shows examples of each type of curve.

We see that single-layer designs are sometimes better than multi-layer designs with limited numbers of layers, but it is always possible to achieve better performance with a multi-layer design, if a large enough number of layers is used. In other words, for some waveforms, an insuf cient number of layers can make loss worse than is achieved for a single layer. We would like to characterize which waveforms can have worse loss with an insuf cient number of layers, and, for those waveforms, how many layers is suf cient to do better than a single-layer design.

The region of amplitude and frequency ratios in which an insuf cient number of layers can make loss worse than is achieved

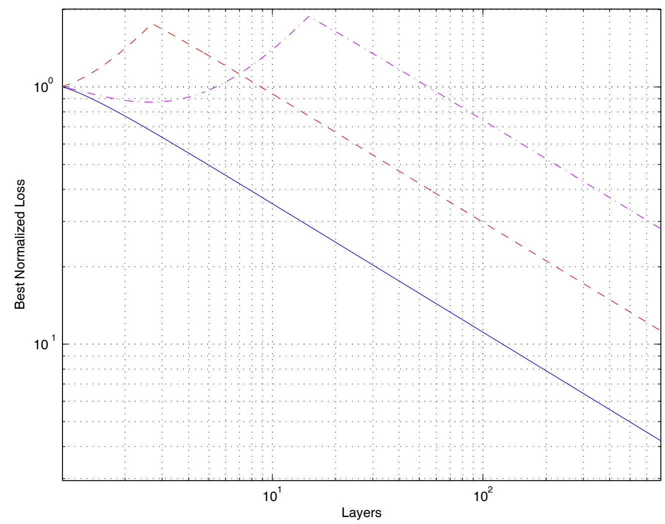


Fig. 12. Normalized best lost versus number of layers for three different waveforms, each comprising two sine waves. Loss is normalized to minimum loss with one layer for each waveform. The solid line curve, which is for a waveform in which amplitude ratio for the two sine waves is 3 and a frequency ratio of 300, has decreasing loss with increasing numbers of layers. The dashed-line curve, which is for an amplitude ratio of 0.5 and a frequency ratio of 700, rst increases but then decreases, eventually getting down below the value for one layer. The dot-dash line curve, which is for an amplitude ratio of .08 and a frequency ratio of 800, begins by decreasing. It then reaches a local minimum, turns around and increases to above 1, and then nally decreases again, eventually getting below the loss at the rst local minimum.

for a single layer is shown in Fig. 13. The number of layers that is suf cient to achieve loss lower than the single-layer loss is indicated by solid contour lines in this region.

Some amplitude and frequency ratio combinations also result in a local minimum of loss with a small number of layers (e.g., the dash-dot curve in Fig. 12). For these waveforms, the multi-layer loss does not become worse than the single-layer loss until above some number of layers. This number of layers is shown by the dot-dash contours in Fig. 13.

It is possible to roughly describe the region of amplitude and frequency ratios in which multi-layer designs with an insuf cient number of layers cannot achieve lower loss than single-layer designs (the region of solid-line contours in Fig. 13) as a triangular region on the plot. The top border of this triangle is near an amplitude ratio of one. (The region spills over this border a little bit, but the contours are rising slowly in this area.) The lower diagonal boundary of the region is described by the amplitude ratio equal to the inverse of the square root of frequency ratio. (In Fig. 13, this boundary is somewhat jagged; this is only an artifact of the limited resolution in the data generated.)

This characterization gives some insight on what types of waveforms in general are likely to to present problems for multi-layer designs with limited numbers of layers. Simplistically, one could expect that it would be dif cult to make multi-layer windings work for waveforms with harmonics whose relative amplitudes and frequencies fall in the triangular region described above. However, since Fig. 13 was generated for waveforms containing only two frequencies, this generalization of the results

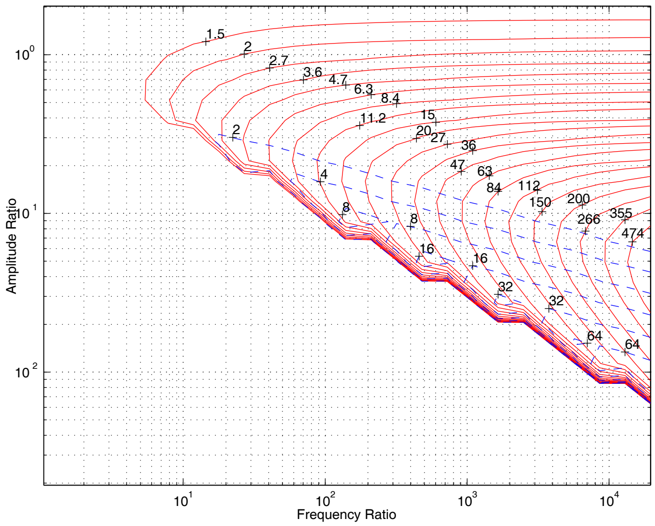


Fig. 13. Contour Plot of amplitude and frequency ratios that result in single-layer loss being greater than multi-layer loss. The solid line curves are labelled with the number of layers needed to have multi-layer loss less than single-layer loss for that amplitude and frequency. The dotted line curve is the number of layers in a multi-layer design when the single-layer loss becomes greater than the multi-layer loss.

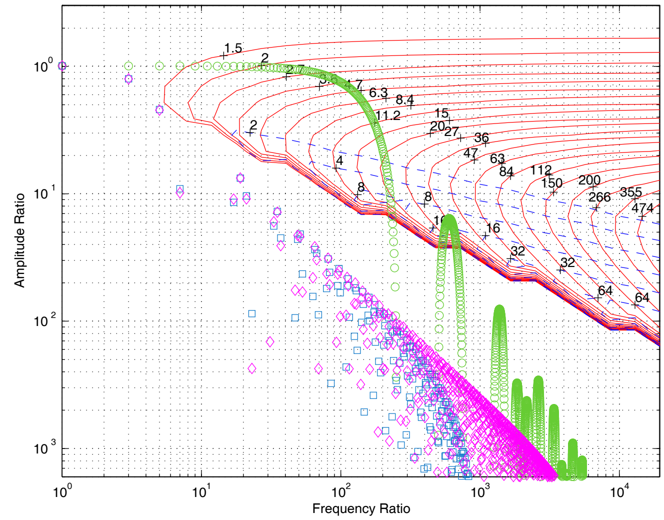


Fig. 14. Harmonics of a Bipolar PWM waveform plotted with the contour lines from Fig. 13. The circular marked harmonics have a duty cycle of 1% and a rise time fraction of 0.1%. The square marked points have a a duty cycle of 26% and a rise time fraction of 0.1%. The diamond marked points have a duty cycle of 26% and a rise time fraction of 0.01%.

is not strictly correct. But we can gain some insight about the types of waveforms that cause dif culties for multi-layer windings by considering the region bounded by an amplitude ratio of one and an amplitude ratio equal to the inverse of the square root of frequency ratio.

A waveform with very high harmonic content, such as a pulse waveform with equal amplitude of all harmonics, would have harmonics lying roughly along the top border of the triangular region. If we consider the fact that the effective harmonic amplitude is higher because of the multiple harmonics, we expect that a multi-layer winding is likely to be effective for a pulse waveform. This is apparent in Fig. 8 in which extreme duty cycles, approaching an ideal pulse waveform, are more easily addressed by multi-layer windings.

Examining other waveforms' Fourier series' asymptotic behavior for high harmonic numbers, we see that $1/f$ for $1/f^2$ behavior is typical. Thus, these asymptotic behaviors fall *below* the triangular region's $1/\sqrt{f}$ lower boundary. However, there are two important limitations that prevent us from drawing hard conclusions from this observation. One is, again, that Fig. 13 was generated for waveforms containing only two frequencies, and so does not directly apply. Another is the lower-frequency harmonic amplitudes do not tend to the asymptotic behavior.

Fig. 14 shows the Fourier series for three bipolar PWM waveforms plotted with the contours in Fig. 13. The harmonics for a 1%-duty-cycle waveform show the similarity of this waveform to a pulse waveform, initially having constant amplitude (amplitude ratio of one), but then dropping because of the nite rise time. The harmonics for a 26% duty cycle show a $1/f$ dependence until, at high frequencies, the amplitudes drop off faster (starting at a lower frequency for slower rise time). What is less clear, however, is the relationship between this plot and the performance of multi-layer designs in reducing loss with these waveforms. It would appear that the 1%-duty-cycle harmonics inside the contour region would make that waveform more dif cult to address with a multi-layer design. However, if we consider the fact that the total harmonic amplitude is higher than the position of any one point plotted, we might consider that each of the sets of points is effectively moved up vertically on the plot. In that case, the points for $D = 1\%$ get moved away from the contour region, whereas the points for $D = 26\%$ get moved into the contour region.

We are only beginning to understand how to think about the effect of the structure of the Fourier series of a waveform on the effectiveness of a multi-layer design. We do not know how to draw solid conclusions from plots such as Fig. 14.

V. Waveforms with Substantial Low-Frequency Content

It is important to note the limitations of the analysis presented thus far. One important assumption is that it is possible to t the optimum thickness in the winding window. For waveforms that consist only of high frequencies, the optimum thickness is usually small, and the optimum design for a given number of layers usually does not ll the window. However, if the frequency is low, or if there is substantial dc or low-frequency current (as is common in recti er and inverter circuits), the theoretical optimum, used in the above calculations, may not t in the winding window.

In general, designs with substantial low-frequency content, or any designs in which full bobbins are optimal, tilt the balance in favor of single-layer windings, for two reasons. Firstly, the overall height of the optimal-thickness winding is likely to be larger for the multi-layer winding. So the multi-layer designs will be further from their optima. Secondly, in a full-bobbin design, packing factor becomes important, and packing factor is typically worse in a multi-layer design, because more layers of insulation are needed.

Thus, we can conclude that single-layer designs are often advantageous in applications that include substantial low-frequency or dc current, although precise quanti cation of when that is the case remains as future work.

VI. Conclusion

The analysis we have presented leads to two main conclusions.

Firstly, we can conclude that, at least for practical power electronics waveforms without substantial dc or low-frequency components, a well designed multi-layer winding with a suf cient number of layers can have lower losses than a single-layer winding. This does not mean that any random multi-layer design is superior it only holds if the winding is properly designed, taking into account all harmonics and optimizing the number of layers and/or the layer thickness.

Secondly, we conclude that for waveforms with strong harmonic content, particularly square waveforms with fast rise times or triangular waveforms with extreme duty cycles, it can often take a large number of layers to effect a signi cant improvement over a single-layer design. For example, a 20% improvement may require four or even ten layers.

Thus, although multi-layer windings can usually provide the lowest loss, the advantage may not always be signi cant, and practical considerations that favor single-layer windings may be more important than the theoretical reduction in loss provided by multi-layer windings. The addition of low-frequency or dc content to a waveform also typically favors single-layer windings.

One particular practical constraint has been analyzed in detail: when a constraint on the thinnest practical layer comes into play. For a sinusoidal waveform and a thickness-to-skin-depth ratio of the thinnest practical layer Δ_{ml} less than 1.49, a multi-layer design is superior, and the loss can be reduced by a factor of $\frac{2}{3}\Delta_{ml}$ with the proper choice of number of layers.

Appendix

To show that above some frequency, the single-layer winding is superior, consider the multi-layer and single-layer cases in Fig. 1. Both windings in these con gurations have equivalent resistances and winding thicknesses much greater than a skin depth. For both con gurations at higher frequencies, the current travels in a layer one skin depth (δ) thick.

The loss in Fig. 1b is given by (1). For Fig. 1a, the current again only travels in a region one skin depth thick in the layer furthest from the gap. In the additional layers, current travels in a region one skin depth thick on both the top and bottom of the layer. Ampere's law can then be used to determine the currents in the windings, and the total loss is equal to the summation of the current in each skin depth squared and multiplied by the winding resistance. This loss value can be expressed as

$$P_{w,ml} = \left(\frac{I_{ac,rms}^2}{p^2}\right) R_\delta \sum_{j=1}^{p} \left(j^2 + (j - 1^2)\right) \qquad (15)$$

which reduces to

$$P_{w,ml} = I_{ac,rms}^2 R_\delta \left(\frac{2p}{3} + \frac{1}{3p}\right) \qquad (16)$$

where R_δ is the ac resistance of a one-skin-depth-thick layer. Single-layer loss can then be approximated by substituting in R_δ for R_{ac} in equation (1). This results in the following single-layer loss equation.

$$P_{w,sl} = I_{ac,rms}^2 R_\delta \qquad (17)$$

Comparing (16) to (17) shows that at high frequencies the single-layer winding has lower loss than the multi-layer winding in the case shown in Fig. 1.

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