Comparison of Loss in Single-Layer and Multi-Layer Windings with a DC Component

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Comparison of Loss in Single-Layer and Multi-Layer Windings with a DC Current Component

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***Abstract* A common technique for reducing losses in high-frequency windings is to use multiple conductor layers, each thin compared to a skin depth. This can be effective, but single-layer windings can also be advantageous for high frequency windings because their ac resistance goes up proportional to the square root of frequency, whereas the ac resistance of multi-layer windings can be proportional to the square of frequency. A general comparison of multi-layer and single-layer windings for applications with signi cant dc current shows that multi-layer windings can be effective in reducing loss in these applications, but achieving lower loss than a single-layer winding may require a large number of layers and may only produce a small improvement in loss.**

I. INTRODUCTION

In magnetic components for modern high-frequency power converters, winding losses can be high because of eddy-current effects. One strategy for reducing eddy-current effects in windings is the use of many layers of thin conductors, such as foil or litz wire. However, single-layer designs also have advantages at high frequency and in some cases a single-layer winding may be better than any feasible multi-layer winding. In a single-layer winding with high-frequency current, the current o ws on the surface in a layer one skin depth deep. Because there are no other layers inducing a eld, the loss depends only on the resistance of the layer where current o ws, and is proportional to the square root of frequency. In contrast, the loss in a multi-layer winding can be proportional to the square of frequency. In the Appendix of [1] it is shown that the performance of any given multi-layer winding degrades at high frequency and eventually becomes worse than that of a single-layer winding. Yet one cannot conclude that a single-layer winding is therefore in general superior for high frequencies, because, for any given high frequency, making the layers thinner in the multi-layer winding can reduce high-frequency losses arbitrarily.

However, there are four arguments in favor of single-layer windings that cannot be dismissed so readily [1]. Firstly, very high frequencies may require thinner conductor layers or thinner wire than is practical. Secondly, the additional effort required for a multi-layer design maybe impractical in comparison to the achievable decrease in loss, particularly if a large number of layers is required. Thirdly, the current waveform may not be a single frequency: it may contain harmonics. Even if the design is optimized for low fundamental-frequency resistance, the harmonics may incur substantial loss if the loss increases in proportion to frequency squared. And nally, the current waveform may contain a large dc component. Even if the design is optimized for low ac resistance, the dc losses may be larger than the ac losses. Previous work in [1] and [2] has addressed three of the cases mentioned above, as brie y summarized in the Appendix. This paper considers the case when the current waveform contains a signi cant dc component, and compares single- and multi-layer windings.

When the current waveform of a magnetic component contains a substantial dc current, signi cant dc-resistance losses are probable. The winding's ac losses can be reduced by con guring conductors in many layers that are each small compared to a skin depth. However, the dc losses may dominate, and achieving low dc resistance with thin layers may be dif cult, both because of the large number of layers that may be needed and also because the space required for insulation between layers hurts the packing factor.

We approach this problem by rst investigating multi-layer designs. If dc losses are suf ciently dominant, the optimal multi-layer design for any given number of layers will ll the winding window. However, with large ac losses, this is not always the case, since the use of thicker layers can degrade high-frequency performance. In Section III-A we investigate the question of when optimized multi-layer designs will ll the winding window, and show that in most practical cases with dc current the optimal multi-layer design does in fact ll the window. Thus, we primarily focus our attention on full-window multi-layer designs, and, in Section III-B compare them to single-layer designs. Before examining these issues in detail, however, we more precisely de ne the problem to be addressed, and review the loss models used, in Section II.

II. PROBLEM DEFINITION AND LOSS MODEL

A. *Parameter De nitions*

We de ne ac resistance in terms of the loss in a winding P_w as

$$R_{ac} = P_w / I_{ac,rms}^2 \tag{1}$$

where $I_{ac,rms}$ is the rms value of the current in the winding. R_{ac} therefore depends on the current waveform. Considering sinusoidal waveforms allows R_{ac} to be de ned as a function of frequency. Because we use (1) and do not consider mutual resistance [3], the analysis in this paper is limited to inductors and two-winding transformers with the same current waveform in both windings (scaled by the turns ratio).

For the purpose of this analysis, the multi-layer and single-layer cases will be abstracted to be those illustrated in Fig. 1: a two-dimensional cross section of a single-layer distributed-gap inductor with a single thick turn of conductor, carrying current I, and the same core with a single-turn conductor divided into p layers of conductor each carrying current I/p. This simple con guration does not directly correspond to any practical situation. In particular, and potentially confusing, is

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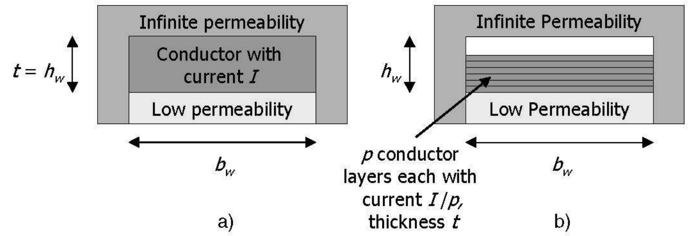

Fig. 1. Basic con gurations analyzed. These are simpli ed models of the real

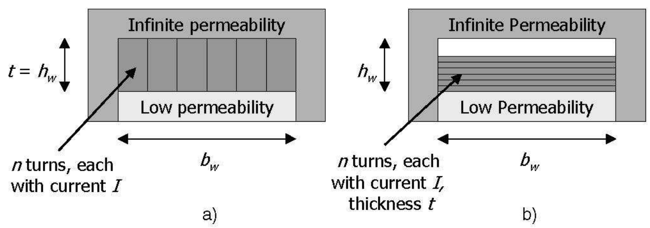

Fig. 2. A comparison of practical single-layer and multi-layer windings (a) Single-layer ($p = 1$) edge-wound foil with six turns ($N = 6$). (b) Barrel-wound foil with the same number of turns ($N = 6$), but with the number of layers equal to the number of turns ($p = N$). All turns are in series in both cases.

that fact that the situation in Fig. 1b does not correspond to a single-turn winding with p layers simply connected in parallel, because in that case the current would not divide evenly between the layers, as we have speci ed for Fig. 1b. However, Fig. 1 can represent many different practical situations, with varying degrees of approximation, as discussed below and in more detail in [1].

B. Application to Practical Winding Types

The con guration in Fig. 1 can be related to many different particular design situations [1]. One example is a comparison of a multi-turn barrel-wound foil winding and an edge-wound winding [4], as shown in Fig. 2 [1]. In this case, the number of layers in the barrel-wound foil, p, is equal to the number of turns, N. By breaking each layer of this design into multiple turns, it is possible to decouple the number of turns from the number of layers, as shown in Fig. 3 [1]. In order to ensure equal current in each conductor segment, this strategy can only be used to obtain a number of layers smaller than the number of turns, such that the conductors in different layers are connected in series. If the layers are in parallel, the currents will not be equal and the analysis does not apply, unless other measures are taken to ensure equal currents in the layers.

In wire windings, litz wire can be used to overcome the dif culties with paralleling conductors, if the strands of ne

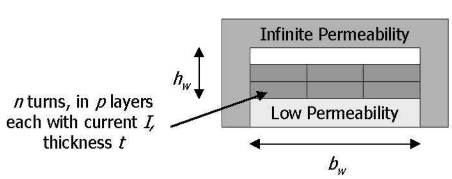

Fig. 3. Modi ed version of Fig. 2b with different numbers of turns and layers: six turns ($N = 6$) and two layers ($p = 2$) All turns are in series.

insulated wire that constitute the litz conductor are twisted together in a true litz pattern such that each strand moves between positions in the bundle, to ensure equal currents in each strand. Litz wire windings do not have neatly structured layers at the strand level, but in most situations this does not signi cantly affect the loss [5], [6].

C. Application to Practical Inductors and Transformers

Fig. 1 shows an inductor with a distributed gap. Although a distributed gap can be a practical way to achieve a one-dimensional eld, it is not very widely used in practice, and the practical cases of most interest are transformer windings and inductors with conventional lumped (discrete) air gaps. The loss analysis is the same for a two-winding transformer with layered construction and an ungapped high-permeability core. Thus, the loss analysis is the same for a simple transformer and the results of analyzing the con guration in Fig. 1 apply directly. For gapped inductors, multiple small gaps can be used to approximate a distributed gap [7]. The results do not apply directly to an inductor with a single lumped gap, unless the gap is spaced well away from the winding[1].

D. Model

For the situations shown in Fig. 1, the eld is one-dimensional, and the Dowell model is an exact solution of Maxwell's equations. The ac resistance factor can be expressed as [8]

$$\frac{R_{ac}}{R_{dc}} = \Delta\left[\frac{\sinh 2\Delta + \sin 2\Delta}{\cosh 2\Delta - \cos 2\Delta} + \frac{2(p^2-1)}{3}\frac{\sinh \Delta - \sin \Delta}{\cosh \Delta + \cos \Delta}\right] \tag{2}$$

where Δ is the ratio of layer thickness to skin depth $\delta = \sqrt{\frac{\rho}{\pi\mu f}}$ where ρ is the conductor resistivity, μ is the conductor permeability and f if the frequency of a sinusoidal current.

As this model is an exact solution of Maxwell's equations, there can be no doubt about its accuracy in representing the ideal situation we are considering. There can, however, be many doubts regarding its accuracy for a particular practical situation. Those issues have been addressed well in the literature, and the accuracy of the model for various situations has been established experimentally (see, e.g., [9], [10]). Although it would be possible to use different models that are more accurate, those models are more accurate because they more carefully model particular situations. Our goal in this paper is to look at the issues in general, rather than for particular situations. Thus, it is more appropriate to use (2). We note, however, that as a result, the data and methods we present should be used only to guide the choice of a general design strategy and to select a winding con guration. Once that con guration has been selected, other models, or nite element analysis, will give more accurate predictions of the actual loss of a particular con guration.

[1]It might seem that the relevant criterion for adequate spacing would be that the distance should be large compared to the gap length, but as explained and shown by simulation in [7], the gap length has little effect and the spacing must not be too small compared to the width of the winding.

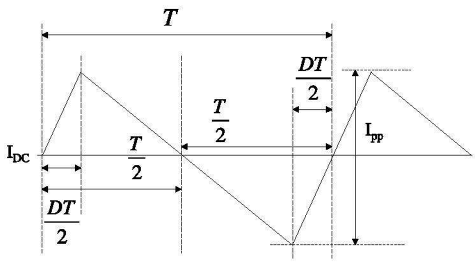

Fig. 4. Triangle waveform with a period T, duty cycle D, dc current I_{dc}, and peak-to-peak current I_{pp}.

III. Designs for Waveforms with High DC Components

Windings with high-frequency current and signi cant dc current (as are common in many power conversion circuits) could be optimized to minimize the high-frequency losses using thin layers, following procedures such as those described in [11], [2], [12], [5], [13], etc. However, optimizing for low ac resistance may degrade dc resistance. If a multi-layer winding is optimized for low dc resistance instead, its ac resistance may end up worse than that of a single-layer winding. So it is not immediately clear whether single- or multi-layer windings are preferred.

To more carefully choose a winding type, we begin by examining the optimal mutli-layer design for a given number of layers. Winding loss is calculated by combining Fourier analysis with the Dowell model (2). Although simpli cations of the Dowell model can often by useful for optimization [11], [13], [12], thick layers needed for low dc resistance typically violate the assumptions of the simpli cations. Thus, we nd the optimal layer thickness (minimizing loss for a given number of layers) through an automatic search process implemented in software. For this case a triangular waveform with period T, duty cycle D, dc current I_{dc}, and peak-to-peak current I_{pp} as shown in Fig. 4 was investigated.

A. Evaluating Window Fill

With a large dc current, it is likely that the optimum design for a given number of layers will be to ll the winding window area, even if that means using thick layers that have high ac resistance. To determine when this is the case, we conducted a systematic set of over 40 000 optimizations for different parameter values.

The optimizations used duty cycles ranging from 0.1 to 0.5, numbers of layers ranging from 1 to 1000, and ripple ratios from 5% to 250%, where ripple ratio is de ned as

$$r_r = \frac{I_{pp}}{I_{dc}} \tag{3}$$

where I_{pp} is the peak-to-peak current and I_{dc} is the dc current in the waveform.

The window height available for the winding is also an important parameter. In order to make our results as general as possible, we do not work in terms of the available window height, nor do we x the conductor resistivity or the frequency. Instead we work in terms of the ratio of the available window height to the skin depth at the fundamental frequency: this is the normalized window height h_{norm}. Similarly, we work with ratios of layer thicknesses to skin depth, Δ, rather than layer thickness explicitly.

We examined a range of normalized window height available from 1 to 1000, meaning window heights from skin depth to 1000 skin depths. This corresponds to, at 10 kHz, heights from 0.67 mm to 668 mm, or at 1 MHz, heights from 67 μm to 67 mm.

The optimization results show that in fact a full bobbin is very often the best choice, providing the lowest possible loss. With signi cant dc current, we consider this the default design, and search for situations in which a full bobbin is *not* the optimum design.

We rst compare duty cycles, and nd that duty cycles closest to 50% more often bene t from a bobbin that is not full, whereas more extreme duty cycles (e.g., 10%, or equivalently, 90%) more universally work best with full bobbins. This difference can be explained by the fact that the harmonic content of the skewed triangle wave with 10% duty cycle is pushed to higher frequencies. In order to reduce the ac loss incurred by these high frequencies, the layers would need to be so thin that the dc loss would be prohibitive, so it is more often preferable to maximize the layer thickness and tolerate the ac losses as the waveform's harmonic content shifts from the fundamental to harmonics.

Thus, in our search for situations in which a full bobbin is *not* the optimum design, we can focus on $D = 0.5$ as the worst case. Fig. 5 shows the space of different possible ripple ratios and normalized heights divided into two regions. In the bottom left region, encompassing most practical parameter values, the full bobbin is the best design for *any* number of layers and for *any* duty cycle. Above the line is a region in which a full bobbin is

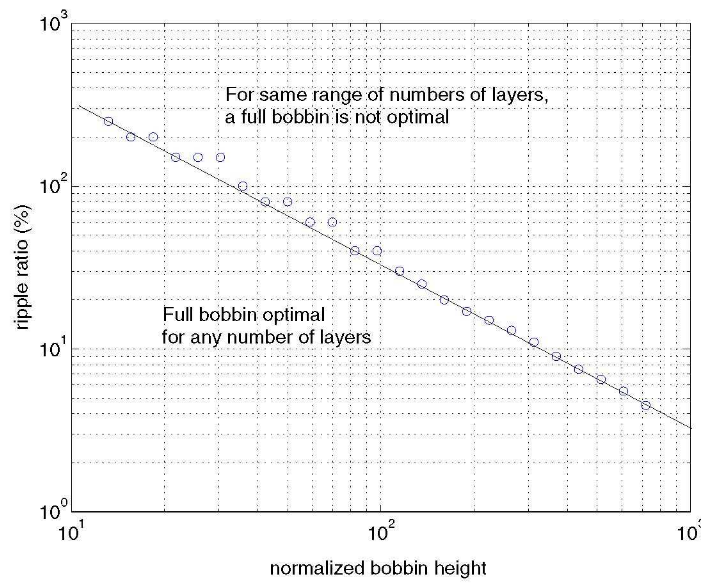

Fig. 5. The space of different possible ripple ratios and maximum winding heights (normalized to a skin depth), divided into two regions: In the lower left region, a full bobbin is always optimal, for any number of layers. In the upper right region, a full bobbin may or may not be optimal, depending on the number of layers. The plot is for a duty cycle of 50%; more extreme duty cycles result in a larger region for which a full bobbin is optimal. (for an example, see Fig. 6. The results apply to any frequency.

not *always* optimal: in this region, at least with 50% duty cycle, a partially lled bobbin can have lower loss than a full bobbin, but only for some speci c region of numbers of layers. Note that Fig. 5 applies at any frequency, due to our use of dimensions normalized to skin depth.

From Fig. 5, we conclude that a full bobbin is the best design for most situations, if (and only if) the waveform contains substantial dc current. For example, at a fairly aggressive ripple ratio of 33%, a full bobbin is *always* optimal up to a normalized bobbin height of 100 (corresponding to 6.7 mm at 1 MHz, or 67 mm at 10 kHz). Even at a ripple ratio of 100%, corresponding to discontinuous conduction, a full bobbin is always optimal up to a normalized bobbin height of 32. It is only above ripple ratios of 200% (as might be used to achieve zero-voltage switching) that the normalized bobbin height for which a full bobbin is always optimal shrinks to below 20. These numbers and Fig. 5 are all for 50% duty cycle; for other duty cycles, the region for which a full bobbin is always optimal expands even further, as shown, for example, in Fig. 6.

On the basis of the results obtained in this section, we focus our further examination on the case of a full window.

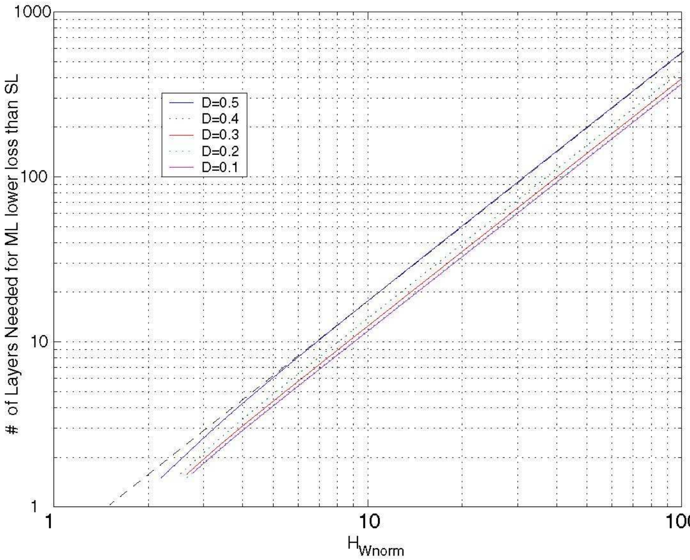

Fig. 7. Number of layers needed to achieve loss in a multi-layer winding equal to that of a single-layer winding, plotted as a function of the winding height normalized to skin depth. Curves for various duty cycles are shown, from 0.5 (top curve) to 0.1 (bottom curve). A straight line (dashed) is shown for comparison. These results apply to any ripple ratio and any frequency.

B. Multi-Layer versus Single-Layer for a Full Window

Given the results above we see that most multi-layer designs for waveforms with dc content lead to a full winding window. Thus, we need only consider full-window designs. But we have not yet compared single-layer designs to multi-layer designs. Assuming similar packing factors for both designs, the difference in loss is determined by the ac resistance. As the number of layers is varied, the ac resistance typically rst increases, and then decreases when the number of layers becomes suf ciently large. An important question is how many layers are needed in order for the loss to be decreased to the point that it is lower than the loss of a single-layer design. Numerical results answering that question are plotted in Fig. 7 which shows the number of layers needed for the loss of a multi-layer design to equal that of a single-layer design, plotted as a function of the normalized window height $h_{w,norm} = h_w/\delta$, where h_w is the height of the window area available for the winding under consideration. For numbers of layers larger than the value plotted, a multi-layer design will be superior, whereas for smaller numbers of layers a single-layer design will be preferred.

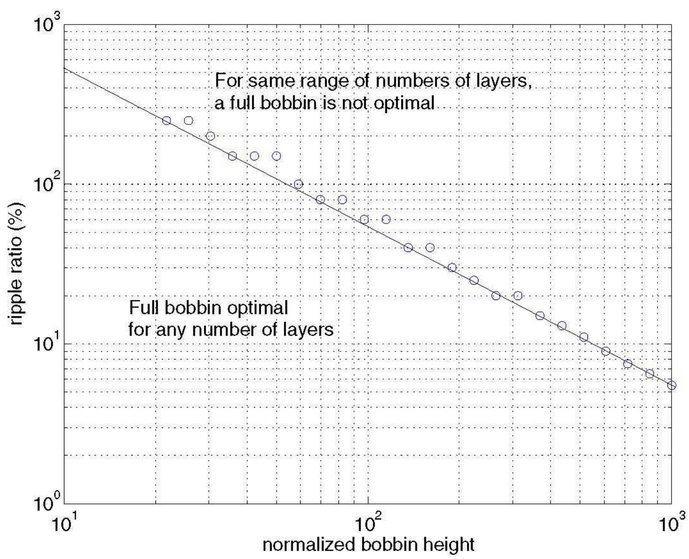

Fig. 6. The space of different possible ripple ratios and maximum winding heights (normalized to a skin depth), divided into two regions: In the lower left region, a full bobbin is always optimal, for any number of layers. In the upper right region, a full bobbin may or may not be optimal, depending on the number of layers. The plot is for a duty cycle of 20%; the results apply to any frequency.

Although the results in Fig. 7 look almost like straight lines on a log scale, a straight line plotted for comparison (dashed) shows that the data can't be modelled quite that simply. However, Fig. 7 includes data that can be used for any application, as long as the optimal designs use a full window, as is the case for most practical designs with dc current. If only a rough approximation is needed, the equation of the dashed line is

$$p_{=} \approx 0.57 h_{w,norm}^{1.5} \qquad (4)$$

where $p_{=}$ is the number of layers needed for the multi-layer loss to equal the single-layer loss; $p > p_{=}$ is needed for the multi-layer design to reduce the loss.

We see that a signi cant number of layers is required to even achieve loss equal to that of a single-layer design. This is especially true for large normalized window heights. Therefore, a single-layer design is often preferred for a waveform with a dc component.

Although Fig. 7 shows which winding type is superior, it does not answer the question of which should be used in practice. In many cases, particularly with a small ripple ratio, the loss of the different designs may be very similar, as shown in Fig. 8. Practical considerations such as manufacturing cost and packing factor may be more important than the ac loss differences.

One can also gain further insight into why the data in Fig. 7 applies to any ripple ratio by observing the difference between the curves for different ripple ratios in Fig. 8. A larger ripple ratio ampli es the difference between the single-layer and multi-layer designs, but whichever one is best, is best independent of the ripple ratio. This is only true under our assumption that the packing factor is the same in both cases.

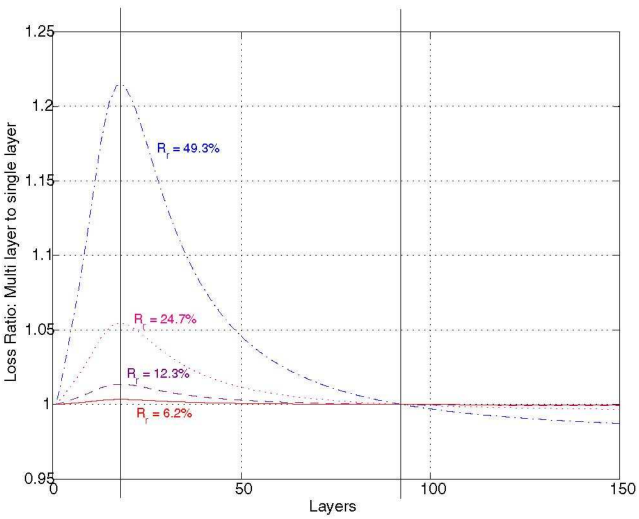

Fig. 8. Ratio of loss for a multi-layer design to loss for a single-layer design, for triangular waveforms with four different ripple ratios, all for duty cycle of 50% and normalized window height limit of 40. The dashed-dot line is for $r_r = 49.3\%$, the dotted line is for $r_r = 24.7\%$, the dashed line is for $r_r = 12.3\%$, and the solid line is for $r_r = 6.17\%$. The solid vertical lines indicate the numbers of layers for maximum loss and for single-layer loss equal to multi-layer loss.

IV. Conclusion

With a current waveform comprising a dc component and triangular ripple, a single-layer winding often has the lowest loss for a given number of layers. A multi-layer winding with many layers can, at least in principle, achieve lower loss, but the number of layers needed can be prohibitive, particularly with a large normalized window height. Note that this is only true for waveforms with a signi cant dc component. As shown in [2] and reviewed in the Appendix loss with ac waveforms can be reduced substantially in many cases by using multi-layer windings even with small numbers of layers.

If a single-layer design is dif cult to construct and the ripple ratio is small, the loss penalty for using a multi-layer design may not be that signi cant. For example, if the ripple ratio is 6.17% and the normalized window height is 40, the loss for an eighteen-layer design, the worst-case number of layers, is only 0.3% higher than that of a single-layer design, as shown in Fig. 8. In this case the loss increase with a multi-layer design over a single-layer design is insigni cant, and a single-layer design would only be justi ed if its cost were lower than the multi-layer design.

Appendix

Multi-Layer and Single-Layer Windings for AC Currents

Although this paper addresses only situations with current waveforms having signi cant dc components, previous work has addressed a wide range of other situations [1], [2]. We summarize some of the main results here; see [2] for further details and explanations of how these results are obtained.

A. Case 1: Constrained thickness

As discussed in [2], when the minimum layer thickness is limited, the improvement possible from a multi-layer design, for sinusoidal waveforms, can be expressed in terms of the thickness-to-skin-depth ratio of the minimum thickness Δ_{min} as

$$\frac{P_{ml}}{P_{sl}} = \frac{2}{3}\Delta_{min}, \tag{5}$$

where P_{ml} and P_{sl} are the power losses in the multi-layer and single-layer windings, respectively. For round wire, this can be written in terms of the minimum wire diameter d_{min} as

$$\frac{P_{ml}}{P_{sl}} = 0.584\left(\frac{d_{min}}{\delta}\right) \tag{6}$$

The achievable thickness to skin depth ratio, Δ_{min}, needs to be less than 1.5 for multiple layers to offer improvement. The optimal number of layers,

$$p_{opt} \approx \frac{3}{\Delta_{min}^2} \tag{7}$$

must be used in order to achieve the loss reduction promised in (5) or (6).

B. Case 2: Constrained number of layers

For a sinusoidal waveform, the following equation can be used to determine the loss reduction possible through an increase in the number of layers [2].

$$\frac{P_{ml}}{P_{sl}} = \frac{1.013}{\sqrt{p}} \tag{8}$$

This calculation is accurate to less than 2% for the number of layers $p \geq 2$ and the ratio of layer thickness to skin depth in the multi-layer design $\Delta_{ml} \leq 1$. The optimal Δ_{ml} for this loss reduction is given by

$$\Delta_{ml_{opt}} = \left(\frac{15}{5p^2-1}\right)^{\frac{1}{4}} \approx \frac{1.3}{\sqrt{p}} \tag{9}$$

C. Case 3: High harmonic content

The requirements for a 20% reduction in loss relative to a single-layer winding for the various waveforms examined in [2] can be brie y summarized as follows:

- For triangular waveforms and triangular and sinusoidal pulse waveforms, both unipolar and bipolar, most reasonable parameters require two layers for a 20% reduction in loss relative to a single-layer winding.
- For rectangular waveforms including bipolar and unipolar PWM waveforms, across most duty cycles, one needs three to six layers for a 20% reduction in loss relative to a single-layer winding.

ACKNOWLEDGMENT

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